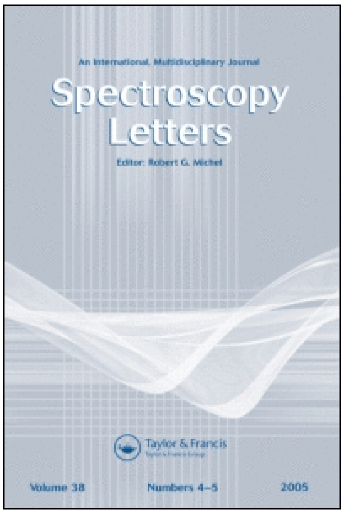

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ON THE THEORY OF BRILLOUIN SCATTERING IN PIEZOELECTRIC SEMICONDUCTORS WITH ACOUSTIC PHONON-CONDUCTION ELECTRON INTERACTION

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The theory of nonresonant Brillouin scattering in anisotropic piezoelectric semiconductors with deformation potential coupling and piezoelectric coupling between excited systems of acoustic phonons and conduction electrons is reviewed. The scattering efficiency is calculated using the appropriate dyadic electromagnetic Green's function. The depletion of the scattered intensity arising from a non phase-matched scattering kinematics and from a spatial exponential decay of the sound amplitude is taken into account. The contributions to the Brillouin scattering from the free-carrier-screened indirect photoelastic effect and from the free-carrier density modulation are expressed in terms of the self-consistent electric field. This field is obtained from a Boltzmann-equation calculation of the effective ac conductivity tensor. The acoustic dispersion of the Brillouin-scattering efficiency is considered, and some possibilities of determining electronic transport properties by means of Brillouin scattering are outlined.

I. INTRODUCTION

The purpose of this review is to present the basic concepts of the semiclassical theory of first-order Brillouin scattering in piezoelectric semiconductors with acoustic phonon-conduction electron interaction [1].

By using the dyadic electromagnectic Green's function appropriate to scattering in optically anisotropic crystals a formal expression for the differential scattering efficiency is obtained. Assuming that the optical and acoustic waves are monochromatic the scattered power is given as a sum of contributions from the spatial Fourier components of the scattering fluctuation. The anisotropic Bragg equations appropriate to a phase-matched scattering kinematics are derived, and the depletion of the scattered intensity due to an imperfect phase match is calculated.

The interaction of the conduction electrons and the acoustic phonons is dominated by the piezoelectric coupling and the deformation potential coupling. Assuming the coupling to be weak the phonon-induced perturbations of the dielectric constant arising from the free-carrier screened indirect photoelastic effect and from the free-carrier density modulation are evaluated in terms of the self-consistent predominantly longitudinal electrostatic field accompanying the acoustic wave. The self-consistent field is in turn expressed by means of the effective frequency- and wave vector-dependent ac conductivity tensor $\overleftrightarrow{\sigma}_{eff}(\Omega, \vec{K})$, and a Boltzmann-equation calculation of $\overleftrightarrow{\sigma}_{eff}(\Omega, \vec{K})$ is outlined. A detailed treatment of the scattering from an exponentially decaying (or growing) acoustic wave, described quantum mechanically by the phonon occupation numbers, is given.

Finally, based on an analysis of the vectorial acoustic dispersion of the Brillouin-scattering efficiency some possibilities of determining $\overleftrightarrow{\sigma}_{eff}(\Omega, \vec{K})$ from phase-matched Brillouin-scattering measurements are discussed.

II. GREEN'S-FUNCTION FORMALISM

In this section we present a formal description of the inelastic scattering of a monochromatic optical wave by a single-frequency fluctuation in the dielectric constant of a semiconducting crystal.

The unperturbed optical wave propagation in a direction given by the unit wave vector $\hat{s}$ can be determined by solving the eigenvalue problem

$$(\overleftrightarrow{1} - \hat{s}\hat{s}) \cdot \vec{e}^{\varphi} = \frac{1}{(n^{\varphi})^2} \overleftrightarrow{\epsilon}_r \cdot \vec{e}^{\varphi} , \qquad (1)$$

where $\overleftrightarrow{1}$ is the unit tensor. In a conducting crystal the complex dielectric tensor takes the form

$$\overleftrightarrow{\epsilon}_r(\omega) = \overleftrightarrow{\epsilon}_r^L(\omega) + i \frac{\overleftrightarrow{\sigma}_0(\omega)}{\epsilon_0 \omega} , \qquad (2)$$

where $\overleftrightarrow{\epsilon}_r^L(\omega)$ is the dielectric tensor of the lattice, and $\overleftrightarrow{\sigma}_0(\omega)$ is the electrical conductivity at the optical frequency $\omega/(2\pi)$. In an absorbing crystal the eigenvalue problem of Eq. (1) leads to solutions in form of plane, damped, elliptically polarized waves described by complex field eigenvectors $\vec{e}^{\varphi}(\hat{s}, \omega)$ and complex refractive indices $n^{\varphi}(\hat{s}, \omega) = n_R^{\varphi} + i n_I^{\varphi}$, $\varphi = 1, 2$.

For a first-order scattering process conservation of energy implies

$$\omega_d = \omega_i \pm \Omega \qquad (3)$$

where ω_i, ω_d, and Ω are the angular frequencies of the incident and diffracted photons and the scattering fluctuation. The plus sign corresponds to an anti-Stokes process and the minus sign to a Stokes process. For phonon-induced scattering one has $\omega = \omega_d \approx \omega_i$.

The amount of electromagnetic radiation generated by the Fourier component $\delta \vec{P}(\vec{r}, \omega)$ of a nonlinear driving polarization can be obtained by solving the inhomogeneous time-independent wave equation for the diffracted field ($\vec{E}_d(\vec{r}, \omega)$)

$$\overleftrightarrow{\alpha}(\vec{\nabla}, \omega) \cdot \vec{E}_d(\vec{r}, \omega) =$$

$$\left\{\left(\frac{c}{\omega}\right)^2 \left[\vec{\nabla}\vec{\nabla} - \nabla^2 \overleftrightarrow{1}\right] - \overleftrightarrow{\epsilon}_r(\omega)\right\} \cdot \vec{E}_d(\vec{r}, \omega) = \frac{1}{\epsilon_0} \delta \vec{P}(\vec{r}, \omega) , \qquad (4)$$

where $\vec{\nabla}$ is the gradient operator. Formally the solution of Eq. (4) is

$$\vec{E}_d(\vec{r}, \omega) = \frac{1}{\epsilon_0} \int_{\vec{r}'} \overleftrightarrow{G}(\vec{r}, \vec{r}') \cdot \delta \vec{P}(\vec{r}', \omega) d\vec{r}' , \qquad (5)$$

where the integration extends over the interaction region. The dyadic Green's function which is the reciprocal of a differential operator can be found by a Fourier transform with respect to space. Introducing the complex vector quantity $\vec{k} = (k_R + i k_I)\hat{s}$ one finds [1, 2]

$$\overleftrightarrow{G}(\vec{R}) = \int_{\substack{-\infty \\ \lim \vec{k}_I \to 0}}^{\infty} \overleftrightarrow{\alpha}^{-1}(\omega, \vec{k}) \exp(i \vec{k}_R \cdot \vec{R}) \frac{d\vec{k}_R}{(2\pi)^3} , \qquad (6)$$

with $\vec{R} = \vec{r} - \vec{r}'$. The reciprocal of the dyadic $\overleftrightarrow{\alpha}(\omega, \vec{k})$ can be expressed in terms of the eigenvectors and eigenvalues for the unperturbed wave propagation. From a decomposition into eigenmodes follows [2]

$$\lim_{\vec{k}_I \to 0} \overleftrightarrow{\alpha}^{-1}(\omega, \vec{k}) = \left(\frac{\omega}{c}\right)^2 \sum_{\varphi=1,2} \frac{\vec{e}^{\varphi}\vec{e}^{\varphi}(n^{\varphi})^2}{k_R^2 - \left(\frac{\omega}{c}\right)^2 \left[(n_R^{\varphi})^2 - (n_I^{\varphi})^2\right] - 2i\left(\frac{\omega}{c}\right)^2 n_R^{\varphi} n_I^{\varphi}} . \tag{7}$$

In Eq. (7) a term containing the eigenvector corresponding to the nonpropagating mode ($n^3 = \infty$) has been omitted since it does not contribute to the Green's function in the asymptotic limit $|R| \to \infty$.

In the following the treatment is limited to semiconducting crystals and it is assumed that the damping of the optical wave can be neglected. Furthermore, effects arising from the free-carrier contribution to the Green's function [1] and resonance effects near the intrinsic absorption edge [3] will not be considered.

Let us consider the inelastic scattering of a photon from the eigenstate θ, $\vec{k}^{\theta}$ into the state φ, $\vec{k}^{\varphi}$. For a plane, undamped linearly polarized wave the electric field is given by

$$\vec{E}_i = \vec{E}^{\theta}(\omega_i, \vec{k}^{\theta}) \exp\left[i(\vec{k}^{\theta} \cdot \vec{r} - \omega_i t)\right] , \tag{8}$$

where the wave vector $\vec{k}^{\theta} = |\vec{k}^{\theta}|\, \hat{s}_i$ and the field eigenvector $\vec{E}^{\theta} = |\vec{E}^{\theta}|\, \hat{e}^{\theta}$ are equal quantities. In the far-field approximation the Green's-function formalism results in the following ratio between the scattered power per unit solid angle around the direction of observation (P^{φ}) and the power in the incident parallel beam (P^{θ})

$$\frac{d P^{\varphi}/d\Omega}{P^{\theta}} = \left(\frac{\omega}{4\pi c}\right)^2 \frac{\left| \hat{e}^{\varphi} \cdot \overleftrightarrow{I}^{\theta,\varphi} \cdot \hat{e}^{\theta} \right|^2}{\chi^{\varphi} n^{\theta} n^{\varphi} \cos \delta^{\theta} \cos \delta^{\varphi}} \frac{V^2}{A}, \qquad (9)$$

where the tensor

$$\overleftrightarrow{I}^{\theta,\varphi} = \frac{1}{V} \int_{\vec{r}'} \exp \left[i(\vec{k}^{\theta} - \vec{k}^{\varphi}) \cdot \vec{r}' \right] \delta \overleftrightarrow{\epsilon}_r(\vec{r}', \Omega) d\vec{r}'. \qquad (10)$$

In Eq. (10) the spatial fluctuation in the relative dielectric tensor, $\delta \overleftrightarrow{\epsilon}_r(\vec{r}, \Omega)$, has been introduced. The unit field eigenvectors ($\hat{e}^{\theta}$, $\hat{e}^{\varphi}$), the Gaussian curvature of the $\omega(\vec{k})$-surface (χ^{φ}), and the refractive indices (n^{θ}, n^{φ}) occurring in Eq. (9) are those associated with the propagation vectors $\vec{k}^{\theta}$ and $\vec{k}^{\varphi}$. The angular deviations between these wave vectors and their related Poynting vectors have been denoted by δ^{θ} and δ^{φ}. The cross sectional area of the incident light beam is A, and the scattering volume is V.

Let us decompose $\delta \overleftrightarrow{\epsilon}_r(\vec{r}, \Omega)$ into its spatial Fourier components, i.e.,

$$\delta \overleftrightarrow{\epsilon}_r(\vec{r}, \Omega) = \frac{1}{V_s} \sum_{\vec{K}} \delta \overleftrightarrow{\epsilon}_r(\Omega, \vec{K}) \exp(\pm i \vec{K} \cdot \vec{r}), \qquad (11)$$

where the sum is over the discrete set of $\vec{K}$ allowed when periodic boundary conditions are introduced over a normalization volume V_s. Combining Eqs. (9) and (11) one obtains

$$\frac{d P^{\varphi}/d\Omega}{P^{\theta}} = \left(\frac{\omega}{4\pi c}\right)^2 \frac{\left| \sum_{\vec{K}} \hat{e}^{\varphi} \cdot \delta \overleftrightarrow{\epsilon}_r (\Omega, \vec{K}) \cdot \hat{e}^{\theta} C_{\vec{K}} \right|^2}{A \chi^{\varphi} n^{\theta} n^{\varphi} \cos\delta^{\theta} \cos\delta^{\varphi}} \left(\frac{V}{V_s}\right)^2 , \qquad (12)$$

with

$$C_{\vec{K}} = \frac{1}{V} \int_{\vec{r}} \exp\left[i (\vec{k}^{\theta} \pm \vec{K} - \vec{k}^{\varphi}) \cdot \vec{r} \right] d\vec{r} . \qquad (13)$$

III. PHASE-MATCHED SCATTERING KINEMATICS

It follows from Eq. (13) that an appreciable amount of scattered radiation occurs only in directions given by the phase-matching condition

$$\vec{k}^{\varphi} = \vec{k}^{\theta} \pm \vec{K} . \qquad (14)$$

In a particle picture this relation expresses the conservation of pseudomomentum in the scattering process. The selection rule of Eq. (14) leads to the anisotropic Bragg equations ($\Omega \ll \omega_i$) [4]

$$\sin\Phi^{\theta} = \frac{\lambda_0}{2 n^{\theta} V_p} \left\{ f + \frac{V_p^2}{\lambda_0^2 f} \left[(n^{\theta})^2 - (n^{\varphi})^2 \right] \right\} , \qquad (15)$$

and

$$\sin \Phi^{\varphi} = \frac{\lambda_0}{2 n^{\varphi} V_p} \left\{ f - \frac{V_p^2}{\lambda_0^2 f} \left[(n^{\theta})^2 - (n^{\varphi})^2 \right] \right\} \quad , \qquad (16)$$

which allow us via the scattering geometry to select the different Ω, $\vec{K}$-components of the dielectric fluctuation. The anisotropic Bragg angles have been denoted by Φ^{θ} and Φ^{φ}, the vacuum wavelength of the incident light by λ_0, and the phase velocity of the Ω, $\vec{K}$-mode by $V_p = V_p(\Omega, \vec{K})$. The frequency of the scattering component is f. In optically isotropic solids Eqs. (15) and (16) are reduced to the normal Bragg law.

If the scattering volume is a rectangular parallellepiped ($V = \prod_i L_i$) one obtains for a scattering process where the degree of mismatch in the wave vectors is given by $\vec{Q} = \vec{k}^{\theta} \pm \vec{K} - \vec{k}^{\varphi}$

$$C_{\vec{K}} = \prod_{i=1}^{3} \frac{\sin\left(\frac{Q_i L_i}{2}\right)}{\frac{Q_i L_i}{2}} \quad , \qquad (17)$$

apart from a phase factor which can be absorbed in $\delta \overleftrightarrow{\epsilon}_r (\Omega, \vec{K})$. Thus, if the light is scattered from a single mode the intensity will be proportional to $|C_{\vec{K}}|^2$.

IV. PHONON-INDUCED PERTURBATION OF THE DIELECTRIC TENSOR

In the following the perturbations in the optical dielectric tensor caused by the spatial Fourier components of an acoustic disturbance are discussed. To describe the acoustic phonon mode we introduce the time-dependent atomic displacement vector given by

$$\vec{u}^{\mu} = u_0^{\mu}(\Omega, \vec{K})\,\hat{\pi}^{\mu} \exp(i\,\vec{K}\cdot\vec{r}) \;, \qquad (18)$$

where $u_0^{\mu}(\Omega, \vec{K})$ is the amplitude of the Fourier component and $\hat{\pi}^{\mu}$ is a unit vector in the direction of polarization of the mode. The index μ labels the different branches in the phonon dispersion relation connecting the angular frequency Ω and the phonon wave vector $\vec{K} = |\vec{K}|\,\hat{\varkappa}$.

In a piezoelectric semiconductor the phonon-induced fluctuations in the dielectric tensor can be decomposed into

$$\delta\overleftrightarrow{\epsilon}_r = \delta\overleftrightarrow{\epsilon}_r^{D} + \delta\overleftrightarrow{\epsilon}_r^{SI} + \delta\overleftrightarrow{\epsilon}_r^{FC} \,. \qquad (19)$$

The first term arising from the fluctuation in the strain tensor and the mean rotation tensor gives the direct photoelastic effect. The second term represents the free-carrier screened indirect photoelastic effect, that is, the succession of the free-carrier screened piezoelectric effect and the electrooptic effect. The third term describes the contribution from the phonon-induced free-carrier density modulation.

A. Direct photoelastic effect

For small strains, the contribution to the inverse dielectric tensor arising from the direct photoelastic effect can be expressed as a linear function of the symmetric and antisymmetric combination of displacement gradients, i.e. [5]

$$(\delta \overleftrightarrow{\epsilon}_r^D)^{-1} = \tfrac{1}{2}\overleftrightarrow{p}^s(\omega)\cdot[\vec{\nabla}\vec{u} + \widetilde{\vec{\nabla}\vec{u}}] + \tfrac{1}{2}\overleftrightarrow{p}^{as}(\omega)\cdot[\vec{\nabla}\vec{u} - \widetilde{\vec{\nabla}\vec{u}}] \,, \qquad (20)$$

where the first term gives the Pöckel contribution to the direct photoelastic effect and the second term gives the rotational contribution. The transpose of the tensor $\vec{\nabla}\vec{u}$ has been denoted by $\widetilde{\vec{\nabla}\vec{u}}$. The photoelastic tensor $\overleftrightarrow{p}^s$ is symmetric upon interchange of the "acoustic" indices, whereas $\overleftrightarrow{p}^{as}$, which can be calculated from the optical dielectric tensor, is antisymmetric in these indices.

When Eq. (20) holds, one obtains from the direct photoelastic effect the contribution [6 - 9]

$$\delta \overleftrightarrow{\epsilon}_r^D(\Omega, \vec{K}) = -iK\overleftrightarrow{\epsilon}_r(\omega)\cdot\overleftrightarrow{p}(\omega)\cdot\hat{\pi}^\mu\hat{\varkappa}\cdot\overleftrightarrow{\epsilon}_r(\omega)u_0^\mu(\Omega, K) \,, \qquad (21)$$

to the Fourier amplitude of the fluctuation of the dielectric tensor. The single photoelastic tensor $\overleftrightarrow{p}(\omega)$ given by $\overleftrightarrow{p}(\omega) = \overleftrightarrow{p}^s(\omega) + \overleftrightarrow{p}^{as}(\omega)$ is neither symmetric nor antisymmetric in the "acoustic" indices.

B. Screened indirect photoelastic effect

An acoustic wave propagating through a piezoelectric semiconductor will be accompanied by a predominantly longitudinal self-consistent electrostatic field arising from the piezoelectric polarization screened by the free carriers. This self-consistent field, $\vec{F}_{sc} \approx |\vec{F}_{sc}|\,\hat{\varkappa}$, causes via the linear electrooptic effect a fluctuation in the inverse dielectric tensor given by [1, 10]

$$(\delta \overleftrightarrow{\epsilon}_r^{SI})^{-1} = \overleftrightarrow{r}(\omega) \cdot \vec{F}_{sc} , \tag{22}$$

where $\overleftrightarrow{r}(\omega)$ is the electrooptic tensor.

The interaction of the free carriers and the acoustic phonons originates mainly from the piezoelectric coupling and the deformation potential coupling. At low frequencies the first coupling mechanism tends to dominate whereas at high frequencies the second is the more important, even in strong piezoelectric crystals.[11, 12]

The critical quantity in determining the free-carrier screening of the electron-phonon interaction, and thus the self-consistent field, is the effective frequency- and wave vector-dependent ac conductivity tensor $\overleftrightarrow{\sigma}_{eff}(\Omega, \vec{K})$ (see section VII)[12]. Combining the constitutive equation for the phonon-induced current, the Maxwell equations, the continuity equation, and the adiabatic piezoelectric equation of state, one obtains for the Fourier component of the self-consistent field the expression

$$\vec{F}_{sc}(\Omega,\vec{K}) = i K u_o^{\mu}(\Omega,\vec{K})\hat{\varkappa} \frac{\dfrac{\hat{\varkappa}\cdot\overleftrightarrow{\sigma}_{eff}(\Omega,\vec{K})\cdot\hat{\varkappa}}{V_p(\Omega,\vec{K})\, q}\,\hat{\varkappa}\cdot\overleftrightarrow{\Xi}\cdot\hat{\pi}^{\mu} - \hat{\varkappa}\cdot\overleftrightarrow{e}\cdot\hat{\pi}^{\mu}\hat{\varkappa}}{\hat{\varkappa}\cdot\left[\overleftrightarrow{\epsilon}^{L}(\Omega) + i\,\dfrac{\overleftrightarrow{\sigma}_{eff}(\Omega,\vec{K})}{\Omega}\right]\cdot\hat{\varkappa}} \tag{23}$$

where $\overleftrightarrow{\Xi}$ is the deformation potential tensor, $\overleftrightarrow{e}$ is the piezoelectric tensor, $\overleftrightarrow{\epsilon}^{L}(\Omega)$ is the low-frequency dielectric tensor of the lattice, and q is the numerical magnitude of the electron charge.

It is seen that the screening of the piezoelectric field is contained in the complex quantity

$$1 + i \frac{\hat{\varkappa} \cdot (\overleftrightarrow{\sigma}_{eff}/\Omega) \cdot \hat{\varkappa}}{\hat{\varkappa} \cdot \overleftrightarrow{\epsilon}^{L} \cdot \hat{\varkappa}} . \qquad (24)$$

Combining Eqs. (22) and (23) one finds that the contribution to $\delta\overleftrightarrow{\epsilon}_r(\Omega, \vec{K})$ from the free-carrier screened indirect photoelastic effect can be written

$$\delta\overleftrightarrow{\epsilon}_r^{SI}(\Omega, \vec{K}) = iKu_o^{\mu}(\Omega, \vec{K})\overleftrightarrow{\epsilon}_r(\omega)\cdot\overleftrightarrow{r}(\omega)\cdot\hat{\varkappa}\cdot\overleftrightarrow{\epsilon}_r(\omega)\times$$

$$\frac{\hat{\varkappa}\cdot\overleftrightarrow{e}(\Omega)\cdot\hat{\pi}^{\mu}\hat{\varkappa} - \dfrac{\hat{\varkappa}\cdot\overleftrightarrow{\sigma}_{eff}(\Omega, \vec{K})\cdot\hat{\varkappa}}{q V_p(\Omega, \vec{K})}\hat{\varkappa}\cdot\overleftrightarrow{\Xi}(\Omega)\cdot\hat{\pi}^{\mu}}{\hat{\varkappa}\cdot\left[\overleftrightarrow{\epsilon}^{L}(\Omega) + i\dfrac{\overleftrightarrow{\sigma}_{eff}(\Omega, \vec{K})}{\Omega}\right]\cdot\hat{\varkappa}} . \qquad (25)$$

In the limit of zero conductivity Eq. (25) is reduced to the well known result [2]

$$\delta\overleftrightarrow{\epsilon}_r^{I}(\Omega, \vec{K}) = iKu_o^{\mu}\overleftrightarrow{\epsilon}_r(\omega)\cdot\overleftrightarrow{r}\cdot\hat{\varkappa}\cdot\overleftrightarrow{\epsilon}_r(\omega)\frac{\hat{\varkappa}\cdot\overleftrightarrow{e}\cdot\hat{\pi}^{\mu}\hat{\varkappa}}{\hat{\varkappa}\cdot\overleftrightarrow{\epsilon}^{L}(\Omega)\cdot\hat{\varkappa}} . \qquad (26)$$

C. Free-carrier density modulation

The buncing of the free carriers induced by the acoustic phonons via the piezoelectric coupling and the deformation potential coupling gives rise to a modulation of the optical dielectric constant. For a nondegenerate solid state plasma one obtains in the low wave vector limit [1]

$$\delta \overleftrightarrow{\epsilon}_r^{FC}(\Omega, \vec{K}) = i \frac{q\, n_1(\Omega, \vec{K})}{\epsilon_0 \omega} \overleftrightarrow{\mu}(\omega) \; , \qquad (27)$$

where $n_1(\Omega, \vec{K})$ is the Fourier amplitude of the free-carrier density modulation, and $\overleftrightarrow{\mu}(\omega)$ is the free-carrier mobility tensor at the optical frequency. For a collision dominated plasma ($\omega\tau_p \ll 1$, τ_p being the electron momentum relaxation time) Eq. (27) takes the form [13, 14]

$$\delta \overleftrightarrow{\epsilon}_r^{FC}(\Omega, \vec{K}) = i \frac{\overleftrightarrow{\sigma}_0(0)}{\epsilon_0 \omega} \frac{n_1(\Omega, \vec{K})}{n_0} \; , \quad \omega\tau_p \ll 1 \qquad (28)$$

where n_0 is the equilibrium free-carrier density, and σ_0 is the dc conductivity tensor. For a collisionless plasma ($\omega\tau_p \gg 1$) one obtains [13, 14]

$$\delta \overleftrightarrow{\epsilon}_r(\Omega, \vec{K}) = - \frac{\overleftrightarrow{\omega_p^2}}{\omega^2} \frac{n_1(\Omega, \vec{K})}{n_0} \; , \quad \omega\tau_p \gg 1 \qquad (29)$$

where $\overleftrightarrow{\omega_p^2}$ is a generalization of the squared angular plasma frequency to the anisotropic case.

The free-carrier density modulation $n_1(\Omega, \vec{K})$ can be expressed in terms of the self-consistent field (Eq. (23)) by combining one of the Maxwell equations, the continuity equation, and the constitutive equation for the ac current. As a result of such a calculation one obtains from the free-carrier density wave the contribution

$$\delta\overleftrightarrow{\epsilon}_r^{FC}(\Omega, \vec{K}) = iKu_o^{\mu}(\Omega, \vec{K}) \frac{\overleftrightarrow{\mu}(\omega)}{\epsilon_0 \omega V_p(\Omega, \vec{K})} \frac{\hat{\varkappa}\cdot\overleftrightarrow{\sigma}_{eff}(\Omega, \vec{K})\cdot\hat{\varkappa}}{\hat{\varkappa}\cdot\left[\overleftrightarrow{\epsilon}^L(\Omega)+i\frac{\overleftrightarrow{\sigma}_{eff}(\Omega, \vec{K})}{\Omega}\right]\cdot\hat{\varkappa}} \times$$

$$\left\{ i\hat{\varkappa}\cdot\overleftrightarrow{e}(\Omega)\cdot\hat{\pi}^{\mu}\hat{\varkappa} + \frac{\Omega}{q V_p(\Omega, \vec{K})} \hat{\varkappa}\cdot\overleftrightarrow{\epsilon}^L(\Omega)\cdot\hat{\varkappa}\hat{\varkappa}\cdot\overleftrightarrow{\Xi}(\Omega)\cdot\hat{\pi}^{\mu} \right\} \tag{30}$$

to the Fourier amplitude of the dielectric fluctuation.

V. EXPONENTIALLY DECAYING LATTICE WAVE

An elastic wave propagating through a crystal will be damped or amplified due to its interaction with its "surroundings". If the interaction is weak the damping (or amplification) becomes exponential. The most important contributions to the frequency- and wave vector-dependent sound attenuation coefficient $\Gamma^{\mu}(\Omega, \vec{K})$ arise from the free-carrier-phonon interaction ($\Gamma^{el\text{-}ph}$), from the elastic anharmonicity ($\Gamma^{ph\text{-}ph}$), from the Brillouin-scattering process (Γ^{Br}), from boundary scattering (Γ^{B}), and from impurities or crystal defects (Γ^{I}). Thus,

$$\Gamma^{\mu}(\Omega, \vec{K}) = \Gamma^{el\text{-}ph} + \Gamma^{ph\text{-}ph} + \Gamma^{Br} + \Gamma^{B} + \Gamma^{I} . \tag{31}$$

In a piezoelectric semiconductor the main contribution to Γ^{el-ph} is due to the piezoelectric coupling and the deformation potential coupling. It is well known that a stimulated phonon emission occurs, i.e., $\Gamma^{el-ph} < 0$ for the piezoelectrically active modes if the component of the free-carrier drift velocity in the direction $\hat{\kappa}$, by application of a sufficiently high external electric field, exceeds the phase velocity of sound in this direction. In a certain region of phonon frequencies this can lead to a net gain of the acoustic wave ($\Gamma^{\mu}(\Omega, \vec{K}) < 0$) [11, 15]. Explicit expressions for Γ^{el-ph} is given in section VI.

A general description of Γ^{ph-ph} for arbitrary $\Omega\tau_p^{ph}$ (τ_p^{ph} being the phonon lifetime) has been developed on basis of a Green's-function method. Relatively simple results can be obtained in the limiting cases $\Omega\tau_{ph}^{p} \ll 1$ (Akhieser loss) and $\Omega\tau_p^{ph} \gg 1$ (Landau-Rumer loss). For the Akhieser mechanism it is assumed that the acoustic wave distorts the lattice leading to a relaxation determined by the third-order elastic constants. According to the Akhieser theory the lattice loss will be proportional to Ω^2. For $\Omega\tau_p^{ph} \gg 1$ the attenuation of transverse acoustic waves is determined by three-phonon scattering processes, while for longitudinal waves four-phonon processes are appropriate. For longitudinal modes uncertainty broadening has to be considered since scattering can occur only with phonons of larger sound velocity. At not too low temperatures the Landau-Rumer loss will be proportional to Ω.

In this work we shall neglect the damping of the optical and acoustical waves arising from the Brillouin-scattering process and only notice that this damping must be taken into account when dealing with intense scattering effects.

For low frequencies or small sample dimensions the boundary scattering can be important especially for off-axis waves.

At very low temperatures or at very high concentrations of impurities or crystal defects, Γ^{I} can play a significant role as a nonelectronic phonon scattering mechanism. For impurity scattering Γ^{I} is proportional to Ω^{4}, whereas it for dislocations is proportional to Ω.

Resolving the time-independent part of the atomic displacement for an exponentially decaying (or growing) wave

$$\vec{u}^{\mu} = u_0^{\mu}(\vec{r}=0,\Omega)\hat{\pi}^{\mu} \exp\left[(i|\vec{K}^{\mu}| - \Gamma^{\mu})\hat{\varkappa}\cdot\vec{r} \right] , \qquad (32)$$

after its spatial Fourier components (denoted by $\vec{q}$) the squared Fourier amplitudes are given by

$$|u_0^{\mu}(\Omega,\vec{q})|^2 = V_s^2 |u_0^{\mu}(\vec{r}=0,\Omega)|^2 \prod_{i=1}^{3} \frac{\sin^2\left(\frac{\Delta K_i^{\mu} a_i}{2}\right) + \sinh^2\left(\frac{\Gamma_i^{\mu} a_i}{2}\right)}{\left(\frac{\Delta K_i^{\mu} a_i}{2}\right)^2 + \left(\frac{\Gamma_i^{\mu} a_i}{2}\right)^2} e^{-\Gamma_i^{\mu} a_i} , \qquad (33)$$

where $\Delta\vec{K}^{\mu} = \vec{K}^{\mu} - \vec{q}$. In the derivation of Eq. (33) it has been assumed that the volume of the solid occupied by the acoustic wave is a rectangular parallellepiped, i.e., $V = \prod_i a_i$ (Integration region $(0 \mid a_i)$). If the integration by choice extends over $\left(-\frac{a_i}{2} \mid \frac{a_i}{2}\right)$ the factor $\exp(-\Gamma_i^\mu a_i)$ vanishes.

To obtain a quantum mechanical description of the lattice vibrations one must make the following replacement for the squared Fourier amplitude of the displacement

$$|u_0^\mu(\Omega, \vec{q})|^2 \rightarrow |\langle n \pm 1 | \vec{u}_q^\mu | n \rangle|^2 = \frac{\hbar\Omega N_{\vec{q}}^{\mu\pm}}{2\rho_0\Omega^2} , \qquad (34)$$

where the plus sign corresponds to a phonon creation (Stokes component) and the minus sign to a phonon annihilation (anti-Stokes component). The occupation number for phonons of wave vector $\vec{q}$ is $N_{\vec{q}}^\mu$, and $N_{\vec{q}}^{\mu\pm}$ is defined by $N_{\vec{q}}^{\mu+} = N_{\vec{q}}^\mu + 1$ and $N_{\vec{q}}^{\mu-} = N_{\vec{q}}^\mu$.

Combining Eqs. (33) and (34) the occupation number for the mode $\vec{q} = \vec{K}$, i.e., $N_{\vec{K}}^\mu$ can be expressed in terms of the squared amplitude $|u_0^\mu(\vec{r} = 0, \Omega)|^2$ of the damped sound wave. Thus,

$$N_{\vec{K}}^{\mu\pm} = \frac{2\rho_0\Omega^2}{\hbar\Omega} v_s^2 \, |u_0^\mu(\vec{r} = 0, \Omega)|^2 \prod_{i=1}^{3} \frac{\sinh^2\left(\frac{\Gamma_i^\mu a_i}{2}\right)}{\left(\frac{\Gamma_i^\mu a_i}{2}\right)^2} e^{-\Gamma_i^\mu a_i} \qquad (35)$$

VI. ELECTRON-PHONON INTERACTION

In a piezoelectric crystal, the time-independent part of the equation of motion for the acoustic lattice vibrations is given by [1]

$$-\rho_0 \Omega^2 \vec{u}^{\mu}(\vec{r}, \Omega) = \vec{\nabla} \cdot \overleftrightarrow{c} \cdot \vec{\nabla} \vec{u}^{\mu}(\vec{r}, \Omega) - \vec{\nabla} \cdot \overleftrightarrow{e} \cdot \vec{F}_{sc}(\vec{r}, \Omega) \quad , \qquad (36)$$

where $\overleftrightarrow{c}$ is the elastic stiffness tensor for constant electric field, and ρ_0 is the mass density of the ion background. For any direction of the acoustic wave vector, specified by $\hat{\kappa}$, the unit polarization vectors $\hat{\pi}^{\mu}$ can be determined approximately by solving the pure elastic eigenvalue problem

$$\rho_0 \left(\frac{\Omega}{K}\right)^2 \hat{\pi}^{\mu} = \overleftrightarrow{c} \cdot \hat{\pi}^{\mu} \hat{\kappa} \cdot \hat{\kappa} \quad . \qquad (37)$$

Below we shall give explicit expressions for the amplitude attenuation coefficient, $\Gamma^{el\text{-}ph}(\Omega, \vec{K})$, and the acoustic phase velocity, $V_p(\Omega, \vec{K})$ in the weak coupling approximation, i.e., for $\Omega / V_p(\Omega, \vec{K}) \gg \Gamma^{el\text{-}ph}(\Omega, \vec{K})$ [10, 13].

Introducing the anisotropic electromechanical coupling constant

$$\tilde{K} = \frac{\hat{\kappa} \cdot \overleftrightarrow{e} \cdot \hat{\pi}^{\mu} \hat{\kappa}}{\left[(\hat{\kappa} \cdot \hat{\pi}^{\mu} \cdot \overleftrightarrow{c} \cdot \hat{\kappa} \cdot \hat{\pi}^{\mu})(\hat{\kappa} \cdot \overleftrightarrow{\varepsilon}^{L} \cdot \hat{\kappa})\right]^{\frac{1}{2}}} = \frac{\tilde{e}}{(\tilde{c}\tilde{\varepsilon}^{L})^{\frac{1}{2}}} \quad , \qquad (38)$$

and the generalizations to the anisotropic case of the effective ac conductivity $\tilde{\sigma}_{eff} = \hat{\kappa} \cdot \overleftrightarrow{\sigma}_{eff} \cdot \hat{\kappa}$, the dc conductivity $\tilde{\sigma}_0 = \hat{\kappa} \cdot \overleftrightarrow{\sigma}_0 \cdot \hat{\kappa}$, the dielectric relaxation frequency $\tilde{\Omega}_c = \tilde{\sigma}_0 / \tilde{\varepsilon}^{L}$, and the deformation potential $\tilde{\Xi} = \hat{\kappa} \cdot \overleftrightarrow{\Xi} \cdot \hat{\pi}^{\mu}$ the anisotropic phase velocity is given by ($\tilde{K}^2 \ll 1$)

$$V_p^{\mu}(\Omega, \vec{K}) = \left(\frac{\tilde{c}}{\rho_0}\right)^{\frac{1}{2}} \left\{ 1 + \frac{\tilde{K}^2}{2} \mathrm{Re}\left(1 + i \frac{\tilde{\sigma}_{eff}}{\tilde{\sigma}_0} \frac{\tilde{\Omega}_c}{\Omega}\right)^{-1} \right\}$$

$$- \frac{\tilde{K}^2}{2} \frac{\tilde{\Xi}/q}{\tilde{e}} \mathrm{Re}\left(\frac{\tilde{\sigma}_{eff}}{1 + i \frac{\tilde{\sigma}_{eff}}{\tilde{\sigma}_0} \frac{\tilde{\Omega}_c}{\Omega}} \right) , \qquad (39)$$

and the amplitude attenuation coefficient as

$$\Gamma^{el\text{-}ph}(\Omega, \vec{K}) = \frac{\tilde{K}^2}{2} \frac{\Omega}{(\tilde{c}/\rho_0)^{\frac{1}{2}}} \left\{ \frac{\tilde{\Xi}/q}{\tilde{e}} \frac{1}{(\tilde{c}/\rho_0)^{\frac{1}{2}}} \times \right.$$

$$\left. \mathrm{Im}\left(\frac{\tilde{\sigma}_{eff}}{1 + i \frac{\tilde{\sigma}_{eff}}{\tilde{\sigma}_0} \frac{\tilde{\Omega}_c}{\Omega}} \right) - \mathrm{Im}\left(1 + i \frac{\tilde{\sigma}_{eff}}{\tilde{\sigma}_0} \frac{\tilde{\Omega}_c}{\Omega}\right)^{-1} \right\} . \qquad (40)$$

VII. CONDUCTIVITY TENSOR $\overleftrightarrow{\sigma}(\Omega, \vec{K})$

A semiclassical approach to the calculation of the ac conductivity tensor uses the Boltzmann equation to determine the electron distribution function $f(\vec{r}, \vec{v}, t)$. For electrons interacting with an acoustic wave $(\Omega, \vec{K})$ in the presence of an external dc electric field $\vec{F}_0$ the Boltzmann equation takes the form [16]

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \frac{\partial f}{\partial \vec{r}} - \frac{q}{m^*} (\vec{F}_0 + \vec{F}_{eff}) \cdot \frac{\partial f}{\partial \vec{v}} = \frac{f - f_0(\vec{v}) - n_1 \frac{\partial f_0(\vec{v})}{\partial n_0}}{\tau_p^{el}(E)} , \qquad (41)$$

where the effective electric field acting on the free electrons in the presence of the acoustic wave can be written

$$\vec{F}_{eff} = \vec{F}_{sc} - \frac{1}{q} \hat{\varkappa}\hat{\varkappa} \cdot \overleftrightarrow{\Xi} \cdot \hat{\pi}^{\mu} K^2 u^{\mu} (\vec{r}, t, \Omega) \quad . \qquad (42)$$

The effective mass of the electron, assumed to be isotropic, has been denoted by m^*, and the energy-dependent electron momentum relaxation time by $\tau_p^{el}(E)$. Treating the electrons as obeying Boltzmann statistics the equilibrium distribution function $f_0(\vec{v})$ is the well known Boltzmann distribution. The term $n_1 \partial f_0/\partial n_0$ arises from the fact that the scattering is local.

To determine the electron distribution function, $f(\vec{r}, \vec{v}, t)$ is decomposed as follows

$$f = f_{dc}(v) + g(v) \exp\left[i(K \cdot r - \Omega t)\right] \quad . \qquad (43)$$

The first term represents the electron distribution function in the presence of the dc field but in the absence of the acoustic wave. Solving the dc part of Eq. (41) one finds to first order in the drift velocity $\vec{v}_d = (-q\tau_p^{el}(E)/m^*)\vec{F}_0$ that $f_{dc} = f_0(\vec{v} - \vec{v}_d)$ i.e. a displaced Boltzmann distribution. Neglecting the non-linear term $(q/m^*)\vec{F}_{eff} \cdot \partial g(v)/\partial \vec{v}$, and taking the direction of the dc field to be along the z-axis the ac part of Eq. (41) turns into a simple inhomogenous first-order differential equation in $g(\vec{v})$.

The phonon-induced current is given by

$$\vec{J}^{el-ph} = -q \int_{-\infty}^{\infty} \vec{v} g(\vec{v}) d\vec{v} = \overleftrightarrow{\sigma}_{eff}(\Omega, \vec{K}) \cdot \vec{F}_{eff} \quad , \qquad (44)$$

where

$$\overleftrightarrow{\sigma}_{eff}(\Omega, \vec{K}) = [\overleftrightarrow{1} - \overleftrightarrow{R}(\Omega, \vec{K})]^{-1} \cdot \overleftrightarrow{\sigma}(\Omega, \vec{K}) . \tag{45}$$

The detailed calculation of the phonon-induced current shows that the conductivity tensor $\overleftrightarrow{\sigma}(\Omega, \vec{K})$ is given by [1]

$$\overleftrightarrow{\sigma}(\Omega, \vec{K}) = \int_{-\infty}^{\infty} \vec{v} \left\{ \int_{-\infty}^{v_z} \frac{\tau_p^{el}(E)}{v_d} \frac{q^2}{m^*} \frac{\partial f_{dc}}{\partial \vec{v}} e^{-a} dv_z' \right\} d\vec{v} , \tag{46}$$

and the tensor $\overleftrightarrow{R}$, arising from the diffusion of the nonuniformly distributed free carriers, by

$$\overleftrightarrow{R}(\Omega, \vec{K}) = \int_{-\infty}^{\infty} \vec{v}\hat{\varkappa} \left\{ \int_{-\infty}^{v_z} \frac{\partial f_0 / \partial n_0}{v_d V_p(\Omega, \vec{K})} e^{-a} dv_z' \right\} d\vec{v} , \tag{47}$$

where

$$a = \int_{v_z'}^{v_z} [i(\vec{K} \cdot \vec{v} - \Omega) + (\tau_p^{el}(E))^{-1}] \frac{\tau_p^{el}(E)}{v_d} dv_z'' . \tag{48}$$

VIII. SCATTERING EFFICIENCY

The scattering from an exponentially decaying (or growing) acoustic wave (Ω, $\vec{K}$, Γ) of polarization type μ between the light polarization states θ and φ can be obtained by combining the results from sections II - VII. Thus, for rectangular volumes of solids and of scattering one obtains for the ratio of scattered to incident powers $\Sigma_\mu^{\theta,\varphi} \equiv (dP^\varphi / d\Omega) / P^\theta$

$$\Sigma_\mu^{\theta,\varphi}(\Omega,\vec{K}) = \left(\frac{\omega}{4\pi c}\right)^2 \frac{\widetilde{M}_\mu^{\theta,\varphi}(\Omega,\vec{K})}{\chi^\varphi n^\theta n^\varphi \cos\delta^\theta \cos\delta^\varphi} \frac{V^2}{A} \left(\frac{\Omega}{V_p(\Omega,\vec{K})}\right)^2 |u_0^\mu(\vec{r}=0)|^2 \times$$

$$\prod_{i=1}^{3} \left[\frac{\sin\left(\frac{Q_i L_i}{2}\right) \sinh\left(\frac{\Gamma_i a_i}{2}\right)}{\left(\frac{Q_i L_i}{2}\right)\left(\frac{\Gamma_i a_i}{2}\right)} \right]^2 e^{-\Gamma_i a_i} . \qquad (49)$$

The matrix element $\widetilde{M}_\mu^{\theta,\varphi}$ is given by

$$\widetilde{M}_\mu^{\theta,\varphi}(\Omega,\vec{K}) = \left| \widetilde{p} - \frac{\widetilde{r}}{\widetilde{\epsilon}}\left(\widetilde{e} - \frac{\widetilde{\Xi}}{q} \frac{\widetilde{\sigma}_{eff}}{V_p}\right) - \frac{\widetilde{\mu}}{\epsilon_0 \omega V_p} \frac{\widetilde{\sigma}_{eff}}{\widetilde{\epsilon}} \left(i\widetilde{e} + \frac{\Omega \widetilde{\epsilon}^L}{V_p} \frac{\widetilde{\Xi}}{q}\right)\right|^2 ,$$

$$(50)$$

with the abbreviations $\widetilde{p} = \hat{e}^\varphi \cdot \overleftrightarrow{\epsilon}_r(\omega) \cdot \overleftrightarrow{p} \cdot \hat{\pi}^\mu \hat{\varkappa} \cdot \overleftrightarrow{\epsilon}_r(\omega) \cdot \hat{e}^\theta$, $\widetilde{r} = \hat{e}^\varphi \cdot \overleftrightarrow{\epsilon}_r(\omega) \cdot \overleftrightarrow{r} \cdot \hat{\varkappa} \cdot \overleftrightarrow{\epsilon}_r(\omega) \cdot \hat{e}^\theta$, and $\widetilde{\mu}(\omega) = \hat{e}^\varphi \cdot \overleftrightarrow{\mu}(\omega) \cdot \hat{e}^\theta$.

Expressed in terms of the occupation number $N_{\vec{K}}^\mu$ the scattering efficiency $\Sigma_\mu^{\theta,\varphi}$ takes the form

$$\Sigma_\mu^{\theta,\varphi}(\Omega,\vec{K}) = \left(\frac{\omega}{4\pi c}\right)^2 \frac{\widetilde{M}_\mu^{\theta,\varphi}}{\chi^\varphi n^\theta n^\varphi \cos\delta^\theta \cos\delta^\varphi} \left(\frac{V}{V_s}\right)^2 \frac{1}{A} \frac{\hbar\Omega N_{\vec{K}}^{\mu\,\pm}}{2\rho_0 V_p^2} \times$$

$$\prod_{i=1}^{3} \frac{\sin^2\left(\frac{Q_i L_i}{2}\right)}{\left(\frac{Q_i L_i}{2}\right)^2} . \qquad (51)$$

IX. ACOUSTIC DISPERSION OF THE SCATTERING EFFICIENCY

It appears from the preceeding analysis that the scattering efficiency $\Sigma_{\mu}^{\theta,\varphi}$ generally is a nontrivial function of the acoustic wave vector, as well as of the incident and scattered optical wave vectors. A quantitative study of the first-order changes of $\Sigma_{\mu}^{\theta,\varphi}$ around a selected wave vector $\vec{K}$ is based on the vectorial acoustic dispersion of the scattering efficiency, $\vec{\Delta}_{\mu}^{\theta,\varphi}$, defined by

$$\vec{\Delta}_{\mu}^{\theta,\varphi} = \frac{\vec{\nabla}_{\vec{K}}\, \Sigma_{\mu}^{\theta,\varphi}}{\Sigma_{\mu}^{\theta,\varphi}} \ . \qquad (52)$$

writing Eq. (52) on the form

$$\vec{\Delta}_{\mu}^{\theta,\varphi} = \frac{1}{\Sigma_{\mu}^{\theta,\varphi}} \; \frac{d\Sigma_{\mu}^{\theta,\varphi}}{d\Omega^{\mu}} \vec{\nabla}_{\vec{K}} \Omega^{\mu} \ , \qquad (53)$$

it is obvious that the vectorial properties of the dispersion effects can be obtained by calculating the acoustic group velocity $\vec{V}_g^{\mu} = \vec{\nabla}_{\vec{K}} \Omega^{\mu}$ of the mode μ, $\vec{K}$. By introducing spherical coordinates (K, θ, φ) the frequency dispersion can be written on the form

$$\hat{\varkappa}_{\vec{K}} \cdot \vec{\Delta}_{\mu}^{\theta,\varphi} = \frac{1}{\Sigma^{\theta,\varphi}} \; \frac{d\Sigma_{\mu}^{\theta,\varphi}}{d\Omega^{\mu}} \left[V_p^{\mu} + K \frac{\partial V_p^{\mu}}{\partial K} \right] , \qquad (54)$$

and the angular dispersion in a direction given by the unit vector $\hat{\varkappa}_t$ ($\hat{\varkappa}_t \cdot \hat{\varkappa}_K = 0$) as follows

$$\hat{\varkappa}_t \cdot \vec{\Delta}_\mu^{\theta,\varphi} = \frac{1}{\Sigma_\mu^{\theta,\varphi}} \frac{d\Sigma_\mu^{\theta,\varphi}}{d\Omega^\mu} \left[\hat{\varkappa}_t \cdot \hat{\varkappa}_\theta \frac{\partial V_p^\mu}{\partial \theta} + \frac{\hat{\varkappa}_t \cdot \hat{\varkappa}_\varphi}{\sin\theta} \frac{\partial V_p^\mu}{\partial \varphi} \right] . \quad (55)$$

The set of mutually perpendicular unit vectors appropriate to spherical coordinates have been denoted by $\hat{\varkappa}_K$, $\hat{\varkappa}_\theta$, and $\hat{\varkappa}_\varphi$.

In strong piezoelectric semiconductors like Zn0 and CdS angular dispersion effects can be significant because of the angular dispersion of the acoustic wave propagation which is induced by elastic anisotropies and by phonon-conduction electron interaction [17, 18].

The basic and numerically appreciable difference between the factor $(d\Sigma_\mu^{\theta,\varphi} / d\Omega^\mu)/\Sigma_\mu^{\theta,\varphi}$ in nonconducting and semiconducting crystals is contained in the dispersion of the matrix element $\widetilde{M}_\mu^{\theta,\varphi}$, i.e. in [19]

$$\frac{1}{\widetilde{M}_\mu^{\theta,\varphi}} \frac{d\widetilde{M}_\mu^{\theta,\varphi}}{d\Omega^\mu} , \quad (56)$$

and in the frequency dispersion of the damping coefficient Γ^{el-ph}.

X. DETERMINATION OF ELECTRONIC TRANSPORT PROPERTIES

In this section some possibilities of determining $\overleftrightarrow{\sigma}_{eff}(\Omega, \vec{K})$ from phase-matched Brillouin-scattering measurements are outlined.

In the general case, the matrix element in Eq. (50) is composed of contributions from the direct photoelastic effect, the screened indirect photoelastic effect, and from the free-carrier density wave. However, by using linearly polarized incident light it is in many cases possible because of (i) different light polarization changes, (ii) anisotropy effects, or (iii) different light frequency dependence to separate the scattering from the three effects.

From the Brillouin-scattering measurements one determines in general only a combination of the real ($\tilde{\sigma}^R_{eff}$) and imaginary ($\tilde{\sigma}^I_{eff}$) part of the appropriate effective ac conductivity. However, since the real and imaginary part of the conductivity are linked via the wave vector-dependent Kramers-Kronig relations a complete determination of $\tilde{\sigma}_{eff}(\Omega, \vec{K})$ can in principle be obtained. When spatial dispersion effects can be neglected the Kramers-Kronig relations take the simple form

$$\tilde{\sigma}^R_{eff}(\Omega) = -\frac{2}{\pi} P \int_0^\infty \frac{\Omega' \tilde{\sigma}^I_{eff}(\Omega')}{\Omega^2 - (\Omega')^2} d\Omega' , \qquad (57)$$

and

$$\tilde{\sigma}^I_{eff}(\Omega) = \frac{2}{\pi} P \int_0^\infty \frac{\tilde{\sigma}^R_{eff}(\Omega')}{\Omega^2 - (\Omega')^2} d\Omega' , \qquad (58)$$

where P denotes the principal value of the integrals.

A. Dominating conduction-electron scattering

In a scattering geometry where the selection rule allows the scattering from the free-carrier density wave it appears from

Eq. (50) that this scattering dominates at long optical wavelengths since $\overleftrightarrow{p}$, $\overleftrightarrow{r}$ and $\overleftrightarrow{\mu}$ are almost independent of frequency in this range. With the definitions $\tilde{\sigma}^N = \tilde{\sigma}^N_R + i\tilde{\sigma}^N_I = (\tilde{\sigma}_{eff}/\tilde{\sigma}_0)(\tilde{\Omega}_c/\Omega)$ one obtains from Eqs. (49) and (50)

$$\tilde{A}_0 \Sigma_\mu^{\theta,\varphi} = \frac{|\tilde{\sigma}^N|^2}{|\tilde{\sigma}^N|^2 + 2\tilde{\sigma}^N_I - 1} , \qquad (59)$$

where the dispersion of the phase velocity and of the damping coefficient induced by $\tilde{\sigma}_{eff}(\Omega, \vec{K})$ are contained in the Ω, $\vec{K}$-dependent quantity $\tilde{A}_0(\Omega, \vec{K})$. The above equation combined with the Kramers-Kronig relations in principle enable one to evaluate the complex conductivity $\tilde{\sigma}_{eff}(\Omega, \vec{K})$.

If the damping of the acoustic wave can be neglected, i.e., for $\Gamma_i a_i \to 0$, the expression for $\tilde{A}_0$ takes a form

$$\tilde{A}_0(\Omega, \vec{K}) = \left(\frac{V_p^\mu}{\Omega}\right)^4 \frac{(4\pi c)^2 A\chi^\varphi n^\theta n^\varphi \cos\delta^\theta \cos\delta^\varphi}{|u_0^\mu(\vec{r}=0)|^2 v^2 |\tilde{\mu}(\omega)|^2} \times$$

$$\frac{1}{\left(\frac{\tilde{e}}{\epsilon_0}\right)^2 + \left(\frac{\Omega \tilde{\epsilon}_r^L \tilde{\Xi}}{q V_p}\right)^2} , \qquad (60)$$

which is independent of $\tilde{\sigma}_{eff}(\Omega, \vec{K})$.

If the scattering takes place from a thermal-equilibrium distribution of phonons one has

$$\left|u_0^{\mu}(\vec{r}=0)\right|^2 = \frac{\hbar\Omega}{\exp\left[(\hbar\Omega)/(k_B T)\right]-1}\ \frac{1}{2\rho_0\Omega^2 V_S^2}, \qquad (61)$$

where k_B is the Boltzmann constant. For microwave phonons at room temperature Eq. (61) simplifies to $|u_0^{\mu}|^2 = k_B T/(2\rho_0\Omega^2 V_S^2)$.

B. Dominating piezoelectric coupling

In the following we consider, under the assumption that $\Gamma = 0$, a few special cases.

When the scattering from the three effects can be separated a measurement of the ratio between the scattering efficiency from the free carriers ($\Sigma_{\mu,FC}^{\theta,\varphi}$) and from the screened indirect effect ($\Sigma_{\mu,SI}^{\theta,\varphi}$) determines the transport properties via an equation of the form

$$|\tilde{\sigma}_{eff}|^2 = \tilde{B}_0 \frac{\Sigma_{\mu,FC}^{\theta,\varphi}}{\Sigma_{\mu,SI}^{\theta,\varphi}}, \qquad (62)$$

where the explicit expression for $\tilde{B}_0$, which apart from a small dispersion of V_p^{μ} is independent of $\tilde{\sigma}_{eff}$, can be obtained easily from Eqs. (49) and (50). If the contributions from SI and FC are separated by the polarization selection rule,scattering with an incident light beam of angular frequency ω simply implies that $\tilde{B}_0(\Omega, \vec{K}) = |\tilde{\mu}|^2/(\epsilon_0 \omega V_p^{\mu} \tilde{r})^2$.

Measurements of the scattering efficiency arising from the screened indirect photoelastic effect allow us to evaluate $\tilde{\sigma}_{eff}$ via the equation

$$1 + |\tilde{\sigma}^N|^2 - 2\tilde{\sigma}_R^N = \tilde{D}_0 \, \Sigma_{\mu,SI}^{\theta,\varphi} \, , \qquad (63)$$

where the explicit conductivity-independent expression for $\tilde{D}_0(\Omega, \vec{K})$ can be obtained from Eqs. (49) and (50).

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